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International Journal of Thermal Sciences 44 (2005) 720–725

International fo Imurol Thermal Sciences

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# Transient natural convection heat transfer by double diffusion from a heated cylinder buried in a saturated porous medium

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Received 7 June 2004; received in revised form 14 February 2005; accepted 22 February 2005

Available online 8 April 2005

#### **Abstract**

This paper presents numerical solutions for the transient natural convection heat transfer by double diffusion from a heated cylinder buried in a saturated porous medium where both, the cylinder and the medium surfaces, are kept at constant uniform temperature and concentration. This situation occurs, for instance, in buried electrical cables, where the ground is the main responsible for the dissipation of heat generated by the electrical cable, where in many cases the cable surface temperature may reach a dangerous limit in a very short time, even before a steady state is attained. Governing equations are expressed in bi-polar coordinates in the stream function formulation and handled numerically by a control volume method. Heat and mass transfer are studied as a function of Rayleigh, Lewis and the buoyancy ratio numbers. 2005 Elsevier SAS. All rights reserved.

*Keywords:* Double diffusion; Transient convection; Control-volume method; Cylinder buried

## **1. Introduction**

Many transport processes presented in the environment occur due to the flow off with a simultaneous occurrence of temperature and concentration gradients. Some oceanographic phenomena like the salt sources are explained by the coupled presence of thermal gradients and saline. There is also an explanation for the dissemination control of pollutants contaminators proceeding from chemical industrial refuse replaceable places and radioactive waste that solves the problem in the area of radioactive spreading out in the ground, in water contamination and in other correlates that still ask for solution [1].

In literature is found some works to solve heat transfer problems in steady state, such as Eckert and Drake [2] and Di Felice and Bau [3] that present a research about heat transfer in a pure conductive medium, in the lack of chemical components dissemination; Bau and Sadhal [4] in-

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vestigate the problem of a semi-infinite cylinder in an homogeneous medium, with mixture frontier conditions (convective), considering an uniform heat transfer coefficient through the cylinder; Schrock et al. [5] studied, including lab experiments, the case of a cylinder buried in a certain depth of a permeable surface, presenting correlation to the temperature distribution when heat transfer starts; Bau [6] presented analytical solutions to natural convection in the cases of Rayleigh numbers smaller than 1, which the convection is induced by a cylinder heated in a saturated and permeable porous medium, where both, cylinder and ground, are kept at constant temperature. Fernandez and Schrock [7] present correlation with the Nusselt number in the case of a cylinder buried in a porous medium saturated with Rayleigh number range between 0.01 and 110. Moya et al. [8] present an experimental analysis of heat and moisture transfer around a heated cylinder surrounded by an unsaturated medium applied to high-voltage electrical power distribution in urban areas which makes use of underground cables.

Chaves [9] realized a study about steady natural convection promoted by double diffusion in saturated porous

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## **Nomenclature**



medium. His study proposes a numerical solution to variations of Rayleigh number (0 to 1000), of Lewis number (0 to 100) and of the buoyancy ratio number ( $-3$  to  $+3$ ), using the control volume method idealized by Patankar [10]. This method has been widely used and its implementation to a bi-polar coordinates system was realized. This paper is a continuation of the study realized by Chaves [9] applied to the case of transient natural convection heat transfer by double diffusion from heated cylinders buried in saturated porous medium. The transient study was motivated by some simplifications realized by Freitas and Prata [11] in the problem of heat and mass transfer around electric cables buried in porous medium.

This paper aims to present numerical solutions for the problem of transient natural convection heat transfer by double diffusion from a heated cylinder buried in a saturated porous medium, exposed to constant uniform temperature and concentration in the cylinder and in the medium surface.

The problem occurs, for example, in electrical conductors when they are buried, where the medium is the main responsible to take off the heat produced by the electrical conductor, through Joule effect. If this medium is no able to take off enough heat, the temperature in the cable surface increases and the electric insulation can be injured. Many times the temperature in the cable's surface can rise up very fast to a dangerous level, even before the process reaches the steady state. The situation characterizes a problem of coupled heat and mass transfer, in transient regime, in saturated porous medium.

In a future step, it will study a case analyzed by Freitas and Prata [11], comparing to its own results.



## **2. Proposition**

Consider an infinite cylinder buried in a saturated porous medium. The cylinder has a radius  $r_1$  and is buried in  $d$  depth from the porous medium superior surface. The cylinder outside cover is kept at  $T_W$  temperature and at  $C_W$  concentration, while the porous medium superior surface is kept at  $T<sub>S</sub>$ temperature and at  $C_S$  concentration, according to Fig. 1. The hypothesis of transient regime and impermeable wall are considered as well.

To obtain the equations that describe the problem, it is assumed [9] that:

(a) the porous medium and the fluid that saturates it are isotropic and homogeneous, the Boussinesq approximation is valid to intensities variation due to changes in both temperature and concentration. It is possible, this way, to express the specific mass according to Bejan [12]:

$$
\rho = \rho_0 \big[ 1 - \beta (T - T_0) - \beta_c (C - C_0) \big] \tag{1}
$$

where  $\beta$  and  $\beta_c$  are the coefficients of thermal and chemical expansion, respectively defined by

$$
\beta = \frac{-1}{\rho_0} \left( \frac{\partial \rho}{\partial T} \right)_p \tag{2}
$$

$$
\beta_c = \frac{-1}{\rho_0} \left( \frac{\partial \rho}{\partial C} \right)_p \tag{3}
$$



Fig. 1. Bi-polar coordinates system.

(b) Darcy law is assumed to describe the fluid flow in porous medium, this way, by Bird et al. [13] is possible to reach the expression

$$
\left(\frac{\mu}{K}\right)\nabla \times \overline{V} = -(\nabla \rho) \times \overline{g}
$$
\n(4)

- (c) the porous medium is rigid and the thermodynamics proprieties (except the density in the buoyancy ratio term) is considered constant,
- (d) there are no chemical reactions and the viscous dissipation are negligible,
- (e) the porous medium and the fluid presents thermodynamic equilibrium.

Based on considered hypothesis for Eqs. (1) and (4), the problem governing equations of transient natural convection heat transfer by double diffusion from a heated cylinder buried in a saturated porous medium can be written for transient regimes and incompressible fluids in the form (Bejan [12] and Chaves [9]):

*Mass conservation*:

$$
\nabla \cdot \overline{V} = 0 \tag{5}
$$

*Moment conservation*:

$$
\left(\frac{\mu}{K}\right)(\nabla \times \overline{V}) = -(\nabla \rho) \times \overline{g}
$$
\n(6)

*Energy conservation*:

$$
\frac{\partial T}{\partial t} + (\overline{V} \cdot \nabla)T = \alpha \nabla^2 T \tag{7}
$$

*Chemical constitution conservation*:

$$
\frac{\partial C}{\partial t} + (\overline{V} \cdot \nabla)C = D\nabla^2 C \tag{8}
$$

To bi-polar coordinates, in the stream function formulation:

*Moment conservation*:

$$
\frac{\partial^2 \Psi}{\partial u^2} + \frac{\partial^2 \Psi}{\partial v^2} = \frac{K \cdot a \rho_0 g}{\mu} \left[ H \left( \beta \frac{\partial T}{\partial u} + \beta_c \frac{\partial C}{\partial u} \right) + G \left( \beta \frac{\partial T}{\partial v} + \beta_c \frac{\partial C}{\partial v} \right) \right]
$$
(9)

where

$$
H = \frac{1 - \cos u \cosh v}{(\cosh v - \cos u)^2} \quad \text{and}
$$
  

$$
G = \frac{\sinh v \sin u}{(\cosh v - \cos u)^2}
$$
(9a)

$$
V_u = \frac{1}{h_v} \frac{\partial \Psi}{\partial v}, \qquad V_v = -\frac{1}{h_u} \frac{\partial \Psi}{\partial u}
$$
(9b)

$$
h_u = h_v = \frac{a}{(\cosh v - \cos u)}\tag{9c}
$$

*Energy conservation*:

$$
\frac{\partial^2 T}{\partial u^2} + \frac{\partial^2 T}{\partial v^2} \n= \frac{1}{\alpha} a^2 F \frac{\partial T}{\partial t} + \frac{1}{\alpha} \left( \frac{\partial \Psi}{\partial v} \frac{\partial T}{\partial u} - \frac{\partial \Psi}{\partial u} \frac{\partial T}{\partial v} \right)
$$
\n(10)

where

$$
F = \frac{1}{(\cosh v - \cos u)^2} \tag{10a}
$$

*Chemical constitution conservation*:

$$
\frac{\partial^2 C}{\partial u^2} + \frac{\partial^2 C}{\partial v^2} \n= \frac{1}{D} a^2 F \frac{\partial C}{\partial t} + \frac{1}{D} \left( \frac{\partial \Psi}{\partial v} \frac{\partial C}{\partial u} - \frac{\partial \Psi}{\partial u} \frac{\partial C}{\partial v} \right)
$$
\n(11)

Eqs.  $(9)$ – $(11)$  of the natural convection heat transfer by double diffusion in saturated porous mediums problem can be written to transient regime and incompressible fluids in bipolar coordinates *(u, v)* presented on Fig. 1 in dimensionless terms, in the stream function formulation by:

$$
\nabla^2 \Psi^* = a^* \left[ \left( H \frac{\partial T^*}{\partial u^*} + G \frac{\partial T^*}{\partial v^*} \right) + N \left( H \frac{\partial C^*}{\partial u^*} + G \frac{\partial C^*}{\partial v^*} \right) \right]
$$
(12)

where

$$
H = \frac{1 - \cos u^* \cosh v^*}{(\cosh v^* - \cos u^*)^2} \quad \text{and}
$$
  

$$
G = \frac{\sinh v^* \sin u^*}{(\cosh v^* - \cos u^*)^2}
$$
(12a)

$$
V_u^* = \frac{1}{h_v} \frac{\partial \Psi^*}{\partial v^*}, \qquad V_v^* = -\frac{1}{h_u} \frac{\partial \Psi^*}{\partial u^*}
$$
(12b)

$$
h_u = h_v = \frac{a^*}{(\cosh v^* - \cos u^*)}
$$
(12c)

$$
\nabla^2 T^* = (a^*)^2 F \frac{\partial T^*}{\partial t^*} + Ra \left( \frac{\partial \Psi^*}{\partial v^*} \frac{\partial T^*}{\partial u^*} - \frac{\partial \Psi^*}{\partial u^*} \frac{\partial T^*}{\partial v^*} \right)
$$
(13)

where

$$
F = \frac{1}{(\cosh v^* - \cos u^*)^2}
$$
 (13a)

$$
\nabla^2 C^* = (a^*)^2 F L e \frac{\partial C^*}{\partial t^*} + Ra L e \left( \frac{\partial \Psi^*}{\partial v^*} \frac{\partial C^*}{\partial u^*} - \frac{\partial \Psi^*}{\partial u^*} \frac{\partial C^*}{\partial v^*} \right)
$$
(14)

where

$$
a^* = \frac{a}{r_1} = \sinh v_1 \tag{14a}
$$

$$
d^* = \frac{d}{r_1} = \cosh v_1 \tag{14b}
$$

$$
T^* = \frac{T - T_s}{T_w - T_s} \tag{14c}
$$

$$
t^* = \frac{\alpha}{r_1^2}t\tag{14d}
$$

$$
C^* = \frac{C - C_s}{C_w - C_s} \tag{14e}
$$

The dimensionless stream function is given by

$$
\Psi^* = \frac{\Psi}{\alpha \, Ra} \tag{14f}
$$

The buoyancy ratio number is given by

$$
N = \frac{\beta_c \Delta C}{\beta \Delta T} \tag{14g}
$$

$$
Le = \frac{\alpha}{D} \tag{14h}
$$

$$
Ra = \frac{Kg\beta\Delta Tr_1}{v\alpha} \tag{14i}
$$

$$
\Delta T = T_w - T_s \tag{14j}
$$
\n
$$
\Delta C = C_w - C_s \tag{14k}
$$

#### **3. Methodology**

To solve the problem numerically, it integrates Eqs. (12)– (14) related to **u** and **v** variables, already described in dimensionless bi-polar coordinates on a generic control volume. Such control volume is described on Fig. 2 and the integration is done following the control volume method formulation developed by Patankar [10] where power law is taken to calculate the flow term through the limits of each internal control volume. Integrating Eq. (12) in the control volume *V CP* on Fig. 2 related to the variables **u** and **v** takes to:

$$
\iint_{VCp} \frac{\partial^2 \Psi^*}{\partial u^2} du \, dv + \iint_{VCp} \frac{\partial^2 \Psi^*}{\partial v^2} du \, dv
$$
  
=  $a^* \bigg[ \iint_{VCp} H \frac{\partial T^*}{\partial u} du \, dv + \iint_{VCp} G \frac{\partial T^*}{\partial v} du \, dv$   
+  $N \bigg( \iint_{VCp} H \frac{\partial C^*}{\partial u} du \, dv + \iint_{VCp} G \frac{\partial C^*}{\partial v} du \, dv \bigg) \bigg]$  (15)

Accepting the power law, suggested by Patankar [10] to calculate the flow terms through the border of each internal control volume, takes to the equation in the form:



Fig. 2. Typical cell of the control volume method.

$$
a_{i,j}\Psi_{i,j}^* = a_{i+1,j}\Psi_{i+1,j}^* + a_{i-1,j}\Psi_{i-1,j}^*
$$
  
+ 
$$
a_{i,j+1}\Psi_{i,j+1}^* + a_{i,j-1}\Psi_{i,j-1}^* + b_{i,j}
$$
 (16)

with

$$
a_{i+1,j} = \frac{\Delta v}{(\delta u)_e} \tag{16a}
$$

$$
a_{i-1,j} = \frac{\Delta v}{(\delta u)_w} \tag{16b}
$$

$$
a_{i,j+1} = \frac{\Delta u}{(\delta v)_n} \tag{16c}
$$

$$
a_{i,j-1} = \frac{\Delta u}{(\delta v)_s} \tag{16d}
$$

where  $\Delta u$ ,  $\Delta v$ ,  $(\delta u)_w$ ,  $(\delta u)_e$ ,  $(\delta v)_n e(\delta v)_s$  are values represented on Fig. 2.

$$
a_{i,j} = a_{i+1,j} + a_{i-1,j} + a_{i,j+1} + a_{i,j-1}
$$
\n
$$
b_{i,j} = a^* \left[ H_{i,j} \left( \frac{\Delta v}{2} \right) \left( T_{i+1,j}^* - T_{i-1,j}^* \right) + G_{i,j} \left( \frac{\Delta u}{2} \right) \left( T_{i,j+1}^* - T_{i,j-1}^* \right) + N \cdot H_{i,j} \left( \frac{\Delta v}{2} \right) \left( C_{i+1,j}^* - C_{i-1,j}^* \right) + N \cdot G_{i,j} \left( \frac{\Delta u}{2} \right) \left( C_{i,j+1}^* - C_{i,j-1}^* \right) \right]
$$
\n
$$
(16f)
$$

To obtain the equations of chemical constituent and energy, it proceeds in a similar way.

For dimensionless terms, in the new coordinate system  $(u, v)$  is established the following boundary and initial conditions, as shown at Fig. 1:

To 
$$
t^* = 0
$$
, it is,  
\n $\Psi^* = 0$ ,  $T^* = C^* = 0$  (initial condition)  
\nTo  $t^* > 0$ , it is,  $u = 0$  and  
\n $v_1 < v \le 0 \Longrightarrow \Psi^* = 0$  (17b)

$$
u = \pi \text{ and } v_1 < v < 0
$$
\n
$$
\implies \Psi^* = 0, \quad \frac{\partial T^*}{\partial v} = \frac{\partial C^*}{\partial v} = 0
$$
\n
$$
v = v_1 \text{ and } 0 \le u \le \pi
$$
\n(17c)

$$
y = v_1
$$
 and  $v \le u \le u$   
\n $\implies \Psi^* = 0, T^* = C^* = 1$ 

*(*over the buried cylinder*)* (17d)

$$
v = 0 \text{ and } 0 \le u \le \pi
$$
  
\n
$$
\implies \Psi^* = 0, \ T^* = C^* = 0
$$
  
\n(over floor surface) (17e)

Boundary conditions, presented at Eq. (17) refers to the flow off domain covering the heated cylinder. The condition  $\Psi^* = 0$  refers to the stagnated fluid. The condition

$$
\frac{\partial T^*}{\partial v} = \frac{\partial C^*}{\partial v} = 0
$$

refers to the mass and fluid absence.

Distinguished equations together with boundary and initial conditions make a coupled system involving stream function, temperature and concentration variables. The numerical solution is treated using the Simple Method purposed by Patankar [10]. To solve this simultaneous mathematical equations that come from distinguish process, is used line-to-line iterative method.

As initial state is considered the following first approximation  $\psi^* = 0$  and  $T^* = C^* = 0$ . To  $t > 0$  is considered the following first approximation  $\psi^* = 0$  and  $T^* = C^* = 1$ (constant temperature and concentration) for the entire domain.

In each process, interaction there was a need for updating  $\psi^*$ ,  $T^*$  and  $C^*$  values and  $\psi^*$  equation was solved three times for each interaction. To reach  $T^*$  and  $C^*$  values it was solved only once by iteration. Such process was widely useful in cases where **Ra** and **Le** are high, that causes stronger convection streams.

The acceptance standard of a solution as converged is based on the maximum error possible inside the whole calculation range. The obtained results convergence was accepted when relative changes in the dependent variables were below  $1.0 \times 10^{-5}$ .

## **4. Results and discussion**

## *4.1. The case analyzed by Bau [6]*

To compare the implemented program for bi-polar coordinates, it was reproduced the Bau conditions [6]: cylinder radius  $r_1 = 0.25$  m; impermeable and isotherm surface of the cylinder; silica floor external to the cylinder (medium size of the corn close to  $2.54 \times 10^{-4}$  m) of permeability  $6 \times 10^{-11}$  m<sup>2</sup>; temperature difference between the cylinder and the floor of  $60^{\circ}$ C; water as saturating liquid, with its properties calculated at  $40^{\circ}$ C; cylinder deepness from the surface floor of 2 m.

Table 1 Nusselt as function of time [s] ( $N = 0$ ,  $Ra = 10.0$ ,  $Le = 1.0$ 

Time $[s]$	<b>Nusselt</b>
1000	2.80
100	2.80
10	2.81
1	3.52
0.1	4.95
0.01	6.26
0.001	11.74
0.0001	28.11
0.00001	49.05
0.000001	54.31

In this conditions the Rayleigh number was  $Ra = 10.0$  $(N = 0, Le = 1.0)$  and the average Nusselt number over the cylinder, according to the author, was 2.80. This Nusselt number was the unique parameter through the analytical solution founded in literature for the comparison. Table 1 presents the comparison in the transient situation. It is possible to verify the concordance with the results provided by Bau [6] for long periods of time. From this confirmation, it numerically simulates the transient condition for several Nusselt and Sherwood numbers, which represent the heat and mass flows in the region of the cylinder, in function of the Lewis and Rayleigh numbers and buoyancy ratio.

## *4.2. Nusselt and Sherwood numbers as function of time*

Considering the multiplicity of dimensionless groups presented in the governing equations and its associated effects, the transient analysis was made sharing the solutions in function of Rayleigh and Lewis numbers and the buoyancy ratio. The results presented were divided:

- (a) flows controlled by heat  $(N = 0)$  what is the class flows basically dominated by buoyancy due to the heating of the cylinder; and
- (b) flow with ascendant buoyancy  $(N = 1)$  where the buoyancy due to the concentration gradient has more influence than the buoyancy coming from the gradient temperature.

As shown on Figs. 3 and 4 for short periods of time, the results show high heat and mass transfer rates farther high Nusselt and Sherwood numbers. Such facts happen due to the boundary and initial conditions imposed to the buried cylinder problem. For long periods of time the results show trustworthy results as well, since the presented values to Nusselt and Sherwood numbers are stead very close to the values found by Chaves [9] and Bau [6] to the steady state. In the case of flows controlled by heat  $(N = 0)$ , the Nusselt and Sherwood numbers are the same for Lewis number and equal to 1 and only Nusselt number was presented. It is possible to notice that on Fig. 2, Nusselt increases with the increasing of Rayleigh and the buoyancy ratio *N* for each



Fig. 3. Nusselt number as function of time, for  $Le = 1$ .



Fig. 4. Nusselt and Sherwood numbers as function of time ( $N = 1.0$ ,  $Ra = 1.0, Le = 10.0$ .

time. An increasing of the Rayleigh number from 1 to 10, takes to an increasing of convective effects, this way taking to high Nusselt and Sherwood numbers, as the concentration field is also influenced by the Rayleigh number. In the flows with ascendant buoyancy  $(N = 1)$ , there are concordant buoyancy forces and as consequence, an intensification of the effects of the natural convection over the flow.

# **5. Conclusions**

The objective of verifying the validity of the implemented program to transient regime to calculate the Nusselt and

Sherwood to several values of Lewis and Rayleigh numbers and the buoyancy ratio was satisfactory concluded. The trustable results obtained to long and short periods of time to the transient natural convection heat transfer by double diffusion from a heated cylinder buried in a saturated and homogeneous porous medium was satisfactory realized.

The comparison with [6] to steady flow show that the transient flow there is a good result when the program is applied for short and long periods of time, it allows to apply the program in many other practical cases.

## **References**

- [1] C.A. Chaves, O.V. Trevisan, Convecção natural duplamente difusiva em torno de um cilindro enterrado num meio poroso saturado, in: Proceedings of the IV ENCIT, Brazilian Congress of Thermal Sciences and Engineering, Rio de Janeiro, Brazil, 1992 (in Portuguese).
- [2] E.R.G. Eckert, R.M. Drake, Analysis of Heat and Mass Transfer, McGraw-Hill, New York, 1972.
- [3] R.F. Di Felice, H.H. Bau, Conductive heat transfer between eccentric cylinders with boundary conditions of the third kind, Trans. Amer. Soc. Mech. Engrg. Ser. C, J. Heat Transfer 105 (1983) 678–680.
- [4] H.H. Bau, S.S. Sadhal, Heat losses from a fluid flowing in buried pipe, Internat. J. Heat. Mass Transfer, Munich (1982) 1621–1629.
- [5] V.E. Schrock, R.T. Fernandez, K. Kesavan, Heat transfer from cylinders embedded in a liquid filled porous medium, in: Proc. of the Int. Heat Transfer Conf., vol. VIII, CT3.6, Paris, 1970.
- [6] H.H. Bau, Convective heat losses from a pipe buried in a semi-infinite porous medium, in: ASME/AICHE National Heat Transfer Conf., vol. 106, Niagara Falls, 1984, pp. 166–175.
- [7] R.T. Fernandez, V.E. Schrock, Natural convection from cylinders buried in a liquid-saturated porous medium, in: Proc. of the Int. Heat Transfer Conf., vol. 2, Munich, 1982, pp. 335–340.
- [8] R.E.S. Moya, A.T. Prata, J.A.B. Cunha Neto, Experimental analysis of unsteady heat and moisture transfer around a heated cylinder buried into a porous medium, Internat. J. Heat Mass Transfer 42 (1999) 2187– 2198.
- [9] C.A. Chaves, Convecção natural duplamente difusiva em torno de um cilindro enterrado num meio poroso saturado, Doctor Thesis, FEC-UNICAMP, Campinas, Brazil, 1990 (in Portuguese).
- [10] S. Patankar, Numerical Heat Transfer and Fluid Flow, Hemisphere, New York, 1980.
- [11] D.S. Freitas, A.T. Prata, Transferência de calor e massa em torno de cabos elétricos enterrados, in: Proceedings of the IV ENCIT, Brazilian Congress of Thermal Sciences and Engineering, Rio de Janeiro, Brazil, 1992 (in Portuguese).
- [12] A. Bejan, Convection Heat Transfer, Wiley, New York, 1984.
- [13] R.B. Bird, W.E. Stewart, E.N. Lightfoot, Transport Phenomena, Wiley, New York, 1976.